Gray-box Monitoring of Hyperproperties

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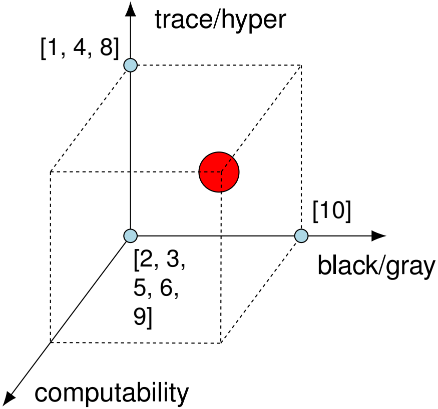
The monitorability cube



Motivation: distributed data minimality

- Distributed data minimality (DDM)
  - privacy property (GDPR)
  - generalization of data minimality to a multi-input setting

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$$\varphi_i \;=\; \forall\pi.\forall\pi'.\exists\tau.\exists\tau'.\,\neg\,\mathrm{same}_i(\pi,\pi') \rightarrow \begin{pmatrix} \mathrm{same}_i(\pi,\tau) \land \mathrm{same}_i(\pi',\tau') \land \\ \mathrm{almost}_i(\tau,\tau') \land \neg\,\mathrm{output}(\tau,\tau') \end{pmatrix}$$

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  - Not black-box monitorable.
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  - Defined over arbitrary domains/datatypes.

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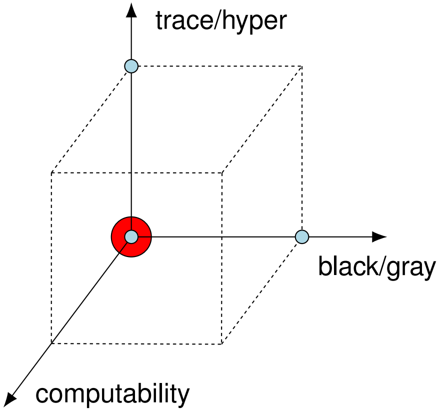
what's going on here?

Trace properties – LTL

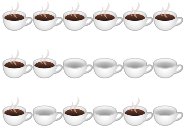


Trace properties – LTL

$\varphi_s = \square$☕ $\qquad \varphi_l = \Diamond$☕ $\qquad \varphi_r = \square\Diamond$☕

Trace properties – LTL

$\varphi_s = \Box$☕ $\qquad \varphi_l = \Diamond$☕ $\qquad \varphi_r = \Box\Diamond$☕



$t_1 = \cdots$	$t_1 \models \varphi_s$	$t_1 \models \varphi_l$	$t_1 \models \varphi_r$
$t_2 = \cdots$	$t_2 \not\models \varphi_s$	$t_2 \models \varphi_l$	$t_2 \not\models \varphi_r$
$t_3 = \cdots$	$t_3 \not\models \varphi_s$	$t_3 \models \varphi_l$	$t_3 \models \varphi_r$

Trace properties – LTL

$\varphi_s = \Box$☕ $\qquad \varphi_l = \Diamond$☕ $\qquad \varphi_r = \Box\Diamond$☕

$t_1 \models \varphi_s$	$t_1 \models \varphi_l$	$t_1 \models \varphi_r$
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$t_3 \not\models \varphi_s$	$t_3 \models \varphi_l$	$t_3 \models \varphi_r$

$\varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \bigcirc\varphi \mid \varphi\,\mathcal{U}\,\varphi \qquad \Diamond\varphi \equiv \text{true}\ \mathcal{U}\ \varphi \qquad \Box\varphi \equiv \neg\Diamond\neg\varphi$

$t \models p$	iff	$p \in t[0]$
$t \models \neg\varphi$	iff	$t \not\models \varphi$
$t \models \varphi_1 \vee \varphi_2$	iff	$t \models \varphi_1$ or $t \models \varphi_2$
$t \models \bigcirc\varphi$	iff	$t[1,..] \models \varphi$
$t \models \varphi_1\,\mathcal{U}\,\varphi_2$	iff	for some i, $t[i,..] \models \varphi_2$ and for all $j < i$, $t[j,..] \models \varphi_1$

Monitoring LTL

$\varphi_s = \square$☕ $\qquad$ $\varphi_l = \lozenge$☕ $\qquad$ $\varphi_r = \square\lozenge$☕

Monitoring LTL

$\varphi_s = \Box$☕ $\qquad$ $\varphi_l = \Diamond$☕ $\qquad$ $\varphi_r = \Box\Diamond$☕

- Observation: the world today at 10am

$u_{10} =$ ☕☕☕☕

Monitoring LTL

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- **Observation:** the world today at 10am

$u_{10} =$ ☕☕☕☕

- **Update:** the world at 11am

$u_{11} =$ ☕☕☕☕☕☕☕☕

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$\varphi_s = \Box$☕ $\varphi_l = \Diamond$☕ $\varphi_r = \Box\Diamond$☕

- Observation: the world today at 10am

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φ_s Is there always coffee?

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φ_s Is there always coffee? $\qquad u_{10} \rightarrow$ **?**

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φ_s Is there always coffee? $\qquad u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗

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  $u_{11} =$ 

φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓

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φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓
φ_r	Is there always eventually coffee?	$u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ **?**

Monitoring LTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), at runtime.

Monitoring LTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

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Definition

A finite observation u permanently satisfies (resp. violates) φ, if every infinite extension of u satisfies (resp. violates) φ:

$$u \models^s \varphi \quad \text{iff} \quad \text{for all } t \in \Sigma^\omega \text{ such that } u \preceq t,\ t \models \varphi$$

$$u \models^v \varphi \quad \text{iff} \quad \text{for all } t \in \Sigma^\omega \text{ such that } u \preceq t,\ t \not\models \varphi$$

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$$u_{11} = \text{☕☕☕☕☕☕☕☕}$$

$u_{11} \not\models^s \Box$☕ $\qquad u_{11} \models^s \Diamond$☕ $\qquad u_{11} \not\models^s \Box\Diamond$☕

$u_{11} \models^v \Box$☕ $\qquad u_{11} \not\models^v \Diamond$☕ $\qquad u_{11} \not\models^v \Box\Diamond$☕

Monitors for LTL

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The monitor M_φ is sound if

$$u \models^s \varphi \quad \text{if} \quad M_\varphi(u) = ✓, \qquad u \models^v \varphi \quad \text{if} \quad M_\varphi(u) = ✗$$

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The monitor M_φ is complete if

$$M_\varphi(u) = ✓ \text{ if } u \models^s \varphi, \quad M_\varphi(u) = ✗ \text{ if } u \models^v \varphi, \quad M_\varphi(u) = \textbf{?} \text{ o/w.}$$

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Fact: every LTL formula has a sound and complete monitor.

Monitorability of LTL formulas

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

$\varphi_r = \Box\Diamond$ 

$u_{11} =$ 

$u_{11} \not\models^{s} \varphi_r$ $\qquad$ $u_{11} \not\models^{v} \varphi_r$

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$$\varphi_r = \Box\Diamond ☕ \qquad u_{11} =$$ 

$$u_{11} \not\models^s \varphi_r \qquad u_{11} \not\models^v \varphi_r$$

Observation: $u \not\models^s \varphi_r$ and $u \not\models^v \varphi_r$ for any u.

Monitorability of LTL formulas

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

$\varphi_r = \Box\Diamond$☕ $u_{11} =$ 

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There's no point in monitoring φ_r!

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$$\varphi_r = \Box\Diamond\text{☕} \qquad\qquad u_{11} = \text{☕☕☕☕☕☕☕☕}$$

$$u_{11} \not\models^s \varphi_r \qquad\qquad u_{11} \not\models^v \varphi_r$$

Observation: $u \not\models^s \varphi_r$ and $u \not\models^v \varphi_r$ for any u.

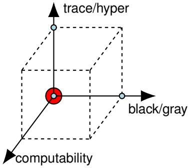
There's no point in monitoring φ_r!

Definition (Pnueli & Zaks 2006)

A formula φ is *(semantically) monitorable* if every observation u has an extension $v \succeq u$, such that either $v \models^s \varphi$ or $v \models^v \varphi$.

LTL – Summary

- Properties defined over individual traces.
  $\Rightarrow$ Properties describe sets of traces.
- Sound and complete monitors can be constructed for any formula.
- Not every formula is monitorable. For example,
  - safety and liveness properties are monitorable,
  - recurrence properties ($\square\lozenge$) are not.



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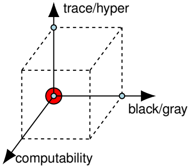


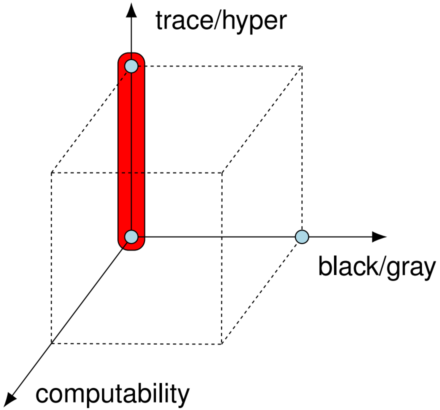
[9] A. Pnueli and A. Zaks. *PSL Model Checking and Run-time Verification via Testers.*, FM'06, Springer, 2006.

[5] Y. Falcone, J-C. Fernandez, and L. Mounier. *What can you verify and enforce at runtime?*, STTT 14(3), 2012.

[8] K. Havelund and D. Peled. *Runtime Verification: From Propositional to First-Order Temporal Logic.* RV'18, Springer, 2018.

. . . and many more!

Hyperproperties – HyperLTL



Hyperproperties – HyperLTL

$\varphi_u = \forall\pi.\forall\tau.\Box(\text{☕}_\pi \rightarrow \text{☕}_\tau)$

$\varphi_a = \forall\pi.\exists\tau.\Box($$_\pi \rightarrow \text{☕}_\tau)$

Hyperproperties – HyperLTL

$\varphi_u = \forall\pi.\forall\tau.\square(☕_\pi \rightarrow ☕_\tau)$ $\qquad$ $\varphi_a = \forall\pi.\exists\tau.\square(○_\pi \rightarrow ☕_\tau)$

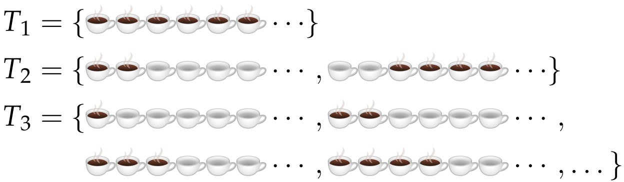
$T_1 = \{☕☕☕☕☕☕\cdots\}$ $\qquad$ $T_1 \models \varphi_u$ $\qquad$ $T_1 \models \varphi_a$

$T_2 = \{☕☕○○○○\cdots, ○○☕☕☕☕\cdots\}$ $\qquad$ $T_2 \not\models \varphi_u$ $\qquad$ $T_2 \models \varphi_a$

$T_3 = \{☕○○○○○\cdots, ☕☕○○○○\cdots,$ $\qquad$ $T_3 \not\models \varphi_u$ $\qquad$ $T_3 \not\models \varphi_a$

$☕☕☕○○○\cdots, ☕☕☕☕○○\cdots, \ldots\}$

Hyperproperties – HyperLTL



$T_1 \models \varphi_u$	$T_1 \models \varphi_a$
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$T_3 \not\models \varphi_u$	$T_3 \not\models \varphi_a$

$$\varphi ::= \forall \pi . \varphi \mid \exists \pi . \varphi \mid \psi \qquad \psi ::= a_\pi \mid \neg \psi \mid \psi \vee \psi \mid \bigcirc \psi \mid \psi \,\mathcal{U}\, \psi$$

$\Pi \models a_\pi$	iff	$a \in \Pi(\pi)[0]$
$\Pi \models \psi_1 \vee \psi_2$	iff	$\Pi \models \psi_1$ or $\Pi \models \psi_2$
$\Pi \models \neg \psi$	iff	$\Pi \not\models \psi$
$\Pi \models \bigcirc \psi$	iff	$\Pi[1..] \models \psi$
$\Pi \models \psi_1 \,\mathcal{U}\, \psi_2$	iff	for some i, $\Pi[i,..] \models \psi_2$, and for all $j < i$ $T, \Pi[j,..] \models \psi_1$

$T, \Pi \models \forall \pi . \varphi$	iff	$T, \Pi[\pi \to t] \models \varphi$ for all $t \in T$
$T, \Pi \models \exists \pi . \varphi$	iff	$T, \Pi[\pi \to t] \models \varphi$ for some $t \in T$
$T, \Pi \models \psi$	iff	$\Pi \models \psi$

Monitoring HyperLTL

$$\varphi_u = \forall \pi . \forall \tau . \Box (\text{☕}_\pi \rightarrow \text{☕}_\tau) \qquad \varphi_a = \forall \pi . \exists \tau . \Box (\text{☕}_\pi \rightarrow \text{☕}_\tau)$$

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- **Observation:** the world today at 10am

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- Update: the world at 11am

  $U_{11} = \{\text{☕☕☕☕☕○○○},\ \text{○○○○☕☕☕☕}\}$

Monitoring HyperLTL

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- Observation: the world today at 10am

  $U_{10} = \{☕☕☕☕\}$

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  $U_{11} = \{☕☕☕☕☕◌◌◌,\ ◌◌◌◌☕☕☕☕,\ ☕☕◌\}$

Monitoring HyperLTL

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φ_u Is there always coffee everywhere at the same time?

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φ_u Is there always coffee everywhere at the same time? $U_{10} \rightarrow$ **?**, $U_{11} \rightarrow$ ✗

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- **Observation:** the world today at 10am

  $U_{10} = \{☕☕☕☕\}$

- **Update:** the world at 11am

  $U_{11} = \{☕☕☕☕☕◡◡◡,\ ◡◡◡◡☕☕☕☕,\ ☕☕◡\}$

φ_u Is there always coffee everywhere at the same time? $U_{10} \rightarrow$ **?**, $U_{11} \rightarrow$ ✗

φ_a Is there always coffee somewhere? $U_{10} \rightarrow$ **?**, $U_{11} \rightarrow$ **?**

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$$U_{11} = \{☕☕☕☕☕○○○,\ ○○○○☕☕☕☕,\ ☕☕○\}$$

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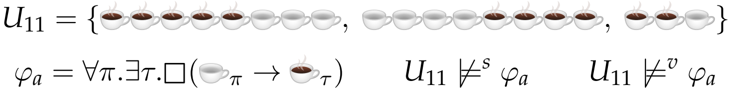
Monitorability of HyperLTL formulas

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation U.



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Observation: $U \not\models^s \varphi_a$ and $U \not\models^v \varphi_a$ for any U.

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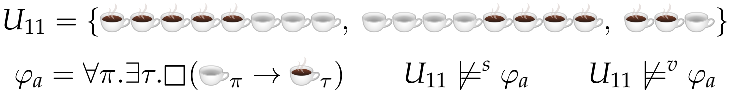

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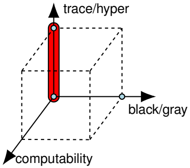
Definition (Agrawal & Bonakdarpour 2016)

A formula φ is *(semantically) monitorable* if every observation U has an extension $V \succeq U$, such that $V \models^s \varphi$ or $V \models^v \varphi$.

HyperLTL – Summary



- Properties defined over sets of traces.
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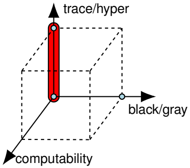

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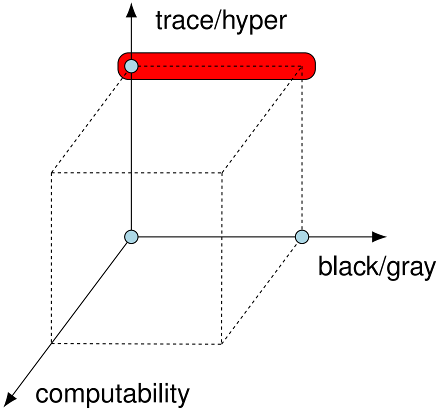
[1] S. Agrawal and B. Bonakdarpour. *Runtime Verification of k-Safety Hyperproperties in HyperLTL.* CSF'16, IEEE CS Press, 2016.

[8] K. Havelund and D. Peled. *Runtime Verification: From Propositional to First-Order Temporal Logic.* RV'18, Springer, 2018.

[7] C. Hahn. *Algorithms for Monitoring Hyperproperties.* RV'19, Springer, 2019.

Gray-box monitoring (of hyperproperties)



Why is φ_a not monitorable?

Theorem

Let $\varphi_a = \forall\pi.\exists\tau.\square($$_\pi \to$ $_\tau)$*. Then* $U \not\models^s \varphi_a$ *and* $U \not\models^v \varphi_a$ *for all* $U \in \mathcal{P}_{\mathit{fin}}(\Sigma^*)$*.*

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$U \not\models^v \varphi_a$ $\;U \preceq \Sigma^\omega$, and $\Sigma^\omega \models \varphi_a$ because ☕☕☕$\cdots \in \Sigma^\omega$;

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This theorem can be generalized to all formulas $\varphi = \forall\pi.\exists\tau.\square P(\pi, \tau)$ where P is

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OK, but let's have a closer look at this proof...

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When monitoring hyperproperties, we'd like to take into account
some information about the system
(gray-box monitoring).

Gray-box monitoring of HyperLTL properties

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation U.

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$$\mathcal{S} = \{T \in \mathcal{P}(\Sigma^\omega) \mid |T| = 3\} \qquad U = \{☕☕◌◌◌,\ ◌◌☕☕◌,\ ☕☕☕☕◌\}$$

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Gray-box monitoring in general

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$$
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Example: $\varphi_a = \forall\pi.\exists\tau.\Box(\text{☕}_\pi \to \text{☕}_\tau)$ $\qquad \mathcal{S} = \{T \in \mathcal{P}(\Sigma^\omega) \mid |T| = 3\}$

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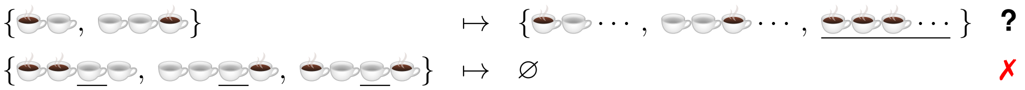
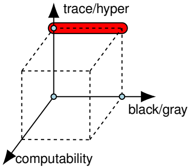
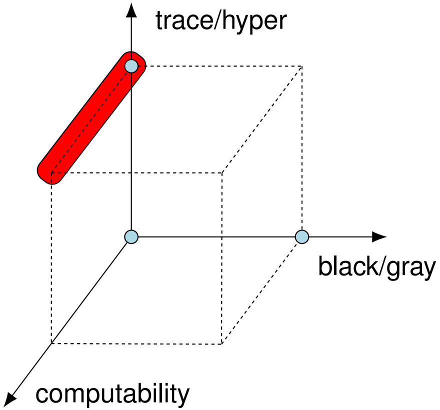


Gray-box monitoring – Summary

- Properties defined over observations (e.g. traces or sets of traces).
  $\Rightarrow$ Properties describe sets of observations.
- Sound and complete monitors can be constructed for some formulas.
  - For example, for formulas without quantifier alternations (as for black-box).
  - But also for $\forall^+\exists^+$-formulas when $\mathcal{S}$ imposes enough constraints.
- Monitorability of formulas depends on set of valid system behaviors $\mathcal{S}$.
  - For example, $\forall^+\exists^+$-properties are monitorable for some choices of $\mathcal{S}$.
  - We will see a more interesting example later...



Undecidable hyperproperties



Monitorability is not existence of monitors

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Example: Let T be some Turing machine.

$$\mathcal{S} = \{t \in \Sigma^\omega \mid t_i = \text{ the state of } T \text{ after } i \text{ steps}\}, \qquad \varphi = \Diamond\text{halt}.$$

Because T is deterministic, either $u \models^s_{\mathcal{S}} \varphi$ or $u \models^v_{\mathcal{S}} \varphi$, for any u in $\mathcal{S}$.

Monitorability is not existence of monitors

A formula φ is semantically gray-box monitorable for a system $\mathcal{S}$ if every observation O has an extension $P \succeq O$ in $\mathcal{S}$, such that $P \models^s_{\mathcal{S}} \varphi$ or $P \models^v_{\mathcal{S}} \varphi$.

A monitor for a property φ and a system $\mathcal{S}$ is a computable function $M_{\varphi,\mathcal{S}} \colon \mathcal{O}\{✓, ✗, ?\}$ that decides a verdict for φ given a finite u.

Observation: Monitorability of φ in $\mathcal{S}$ does not guarantee the existence of a sound and complete monitor $M_{\varphi,\mathcal{S}}$.

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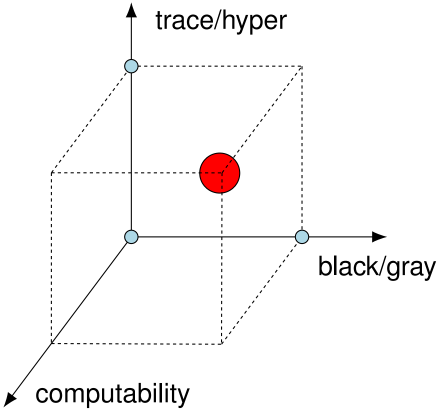
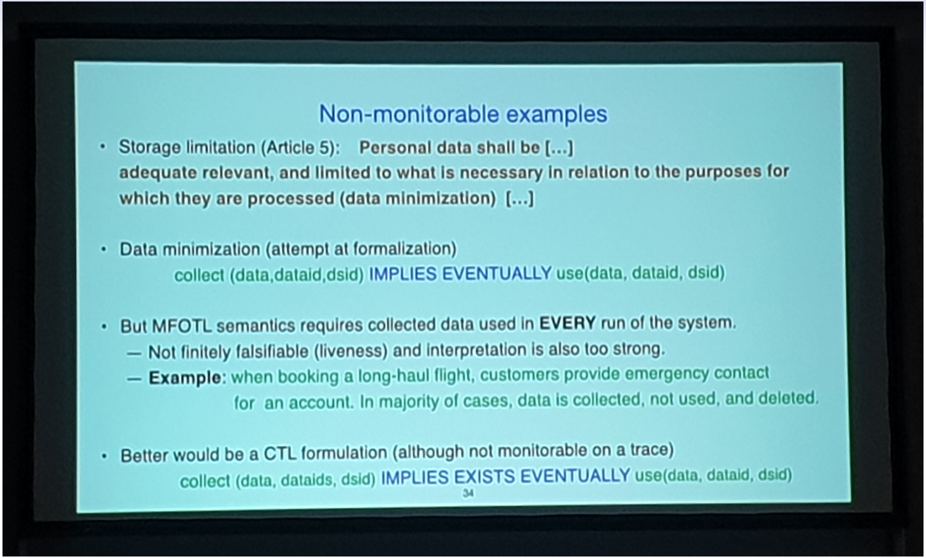
$\Rightarrow$ *there is a sound monitor $M_{\varphi,\mathcal{S}}$ that only answers $\checkmark$ or* ***?****!*

Case study: distributed data minimality





Slide by David Basin, *Can we Verify GDPR Compliance?*, RV’19 keynote.

Case study: distributed data minimality

- Distributed data minimality (DDM)
  - privacy property (GDPR)
  - generalization of data minimality to a multi-input setting
  - $\forall\forall\exists\exists$-hyperproperty

$$\varphi_i \;=\; \forall\pi.\forall\pi'.\exists\tau.\exists\tau'.\, \neg\,\mathrm{same}_i(\pi,\pi') \rightarrow \begin{pmatrix} \mathrm{same}_i(\pi,\tau) \wedge \mathrm{same}_i(\pi',\tau') \wedge \\ \mathrm{almost}_i(\tau,\tau') \wedge \neg\,\mathrm{output}(\tau,\tau') \end{pmatrix}$$

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Yet, we have a monitor...

here's how...

Distributed data minimality

Definition (Antignac, Sands & Schneider, 2017)

A function f is distributed data-minimal (DDM) if, for all input positions k and all $x, y \in I_k$ such that $x \neq y$, there is some $z \in I$, such that $f(z[k \mapsto x]) \neq f(z[k \mapsto y])$.

Distributed data minimality

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$$\varphi_{\mathsf{dm}} \;=\; \bigwedge_{i=1}^{n} \varphi_i, \qquad \Sigma_f^{\#} = \{(x,y) \mid f(x) = y\}, \qquad \mathcal{S}_f = \mathcal{P}(\Sigma_f^{\#})$$

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- Set of observable behaviors $\mathcal{O} = \Sigma_f^{\#}$ are valid function applications.

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Using the generalized framework

- Set of observable behaviors $\mathcal{O} = \Sigma_f^{\#}$ are valid function applications.
- Not black-box monitorable, but gray-box monitorable (thanks to $\mathcal{S}$).

A sound monitor for distributed data minimality

$$\varphi_i \;=\; \forall\pi.\forall\pi'.\exists\tau.\exists\tau'.\ \neg\,\mathrm{same}_i(\pi,\pi') \rightarrow \begin{pmatrix} \mathrm{same}_i(\pi,\tau) \wedge \mathrm{same}_i(\pi',\tau') \wedge \\ \mathrm{almost}_i(\tau,\tau') \wedge \neg\,\mathrm{output}(\tau,\tau') \end{pmatrix}$$

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We build a monitor

$$M_{\text{dm}}(U) = \begin{cases} \textbf{?} & \text{if } f(u_{in}) \neq u_{out} \text{ for some } u \in U, \\ \textbf{?} & \text{if } \bigwedge_{i=1}^{n} \bigwedge_{u,u' \in U} N_{f,i}(\mathrm{proj}_i(u_{in}), \mathrm{proj}_i(u'_{in})) \neq \text{✗}, \\ \text{✗} & \text{otherwise.} \end{cases}$$

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using an oracle $N_{f,i}(x,y)$ (implemented as symbolic execution + SMT solver):

$$N_{f,i}(x,y) = \begin{cases} ✓ \text{ or } \textbf{?} & \text{if } \exists z \in I. f(z[i \mapsto x]) \neq f(z[i \mapsto y]), \\ ✗ \text{ or } \textbf{?} & \text{otherwise.} \end{cases}$$

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The monitor is sound but not complete.

Try it out!

https://github.com/sstucki/minion/

Thank you!

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IOWA STATE UNIVERSITY

Checkout the `minion` monitor for data minimality

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