Gray-box Monitorability of Hyperproperties

The Case of Data Minimality

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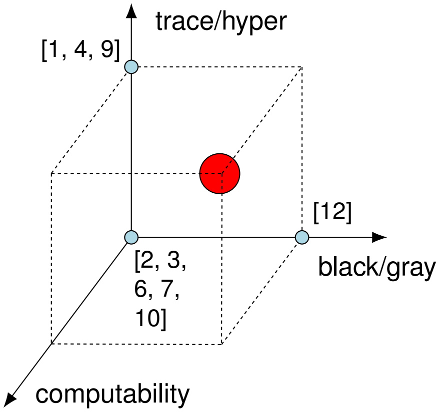
The monitorability cube



Motivation: distributed data minimality

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    - privacy property (GDPR)
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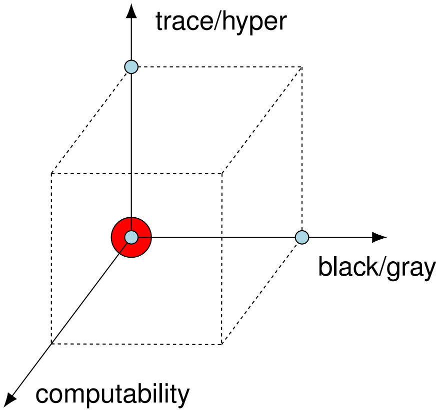
Yet, we have a monitor…

what's going on here?

Trace properties – LTL



Monitoring LTL

$\varphi_s = \square$☕ $\varphi_l = \Diamond$☕ $\varphi_r = \square\Diamond$☕

Monitoring LTL

$\varphi_s = \Box$☕ $\qquad \varphi_l = \Diamond$☕ $\qquad \varphi_r = \Box\Diamond$☕

- Observation: the world today at 10am

$u_{10} =$ ☕☕☕☕

Monitoring LTL

$$\varphi_s = \Box ☕ \qquad \varphi_l = \Diamond ☕ \qquad \varphi_r = \Box\Diamond ☕$$

- Observation: the world today at 10am

$u_{10} = ☕☕☕☕$

- Update: the world at 11am

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φ_s Is there always coffee?

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φ_s Is there always coffee? $\qquad u_{10} \rightarrow$ **?**

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φ_s Is there always coffee? $\qquad u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗

Monitoring LTL

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φ_s Is there always coffee? $\quad u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗

φ_l Is there eventually coffee? $\quad u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓

Monitoring LTL

$\varphi_s = \square$☕ $\varphi_l = \diamond$☕ $\varphi_r = \square\diamond$☕

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φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?**, $u_{11} \rightarrow$ ✗
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Monitoring LTL

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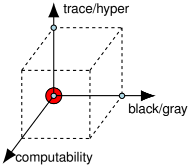
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓
φ_r	Is there always eventually coffee?	$u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ **?**

A monitor for a property φ is a computable function $M_\varphi \colon \Sigma^* \rightarrow \{$✓, ✗, **?**$\}$ that decides whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

LTL – Summary



- Properties defined over individual traces.
  $\Rightarrow$ Properties describe sets of traces.
- Perfect monitors can be constructed for any formula.
- Not every formula is monitorable. For example,
  - safety and liveness properties are monitorable,
  - recurrence properties ($\Box\Diamond$) are not.

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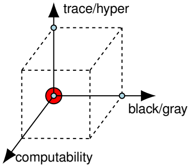

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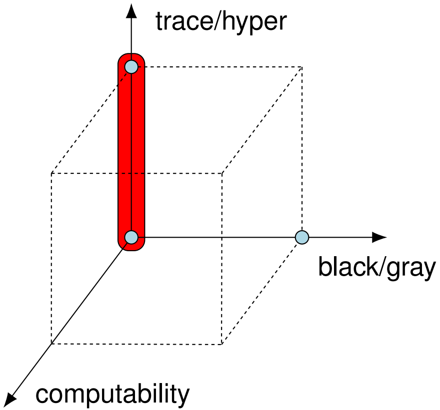
[6] Y. Falcone, J-C. Fernandez, and L. Mounier. *What can you verify and enforce at runtime?*, STTT 14(3), 2012.

[9] K. Havelund and D. Peled. *Runtime Verification: From Propositional to First-Order Temporal Logic.* RV'18, Springer, 2018.

... and many more!

Hyperproperties – HyperLTL



Hyperproperties – HyperLTL

$$\varphi_u = \forall \pi . \forall \tau . \Box (\text{☕}_\pi \rightarrow \text{☕}_\tau) \qquad \varphi_a = \forall \pi . \exists \tau . \Box (\text{☕}_\pi \rightarrow \text{☕}_\tau)$$

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$$T_1 = \{\text{☕☕☕☕☕☕}\cdots\}$$

$$T_2 = \{\text{☕☕◯◯◯◯}\cdots,\ \text{◯◯☕☕☕☕}\cdots\}$$

$$T_3 = \{\text{☕◯◯◯◯◯}\cdots,\ \text{☕☕◯◯◯◯}\cdots,\ \text{☕☕☕◯◯◯}\cdots,\ \text{☕☕☕☕◯◯}\cdots,\ \ldots\}$$

$$T_1 \models \varphi_u \qquad T_1 \models \varphi_a$$

$$T_2 \not\models \varphi_u \qquad T_2 \models \varphi_a$$

$$T_3 \not\models \varphi_u \qquad T_3 \not\models \varphi_a$$

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$$T_1 = \{☕☕☕☕☕☕\cdots\}$$

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$T_1 \models \varphi_u$ $\qquad$ $T_1 \models \varphi_a$

$T_2 \not\models \varphi_u$ $\qquad$ $T_2 \models \varphi_a$

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$$\varphi ::= \forall\pi.\varphi \mid \exists\pi.\varphi \mid \psi \qquad\qquad \psi ::= a_\pi \mid \neg\psi \mid \psi \vee \psi \mid \bigcirc\psi \mid \psi\,\mathcal{U}\,\psi$$

$\Pi \models a_\pi$	iff	$a \in \Pi(\pi)[0]$
$\Pi \models \psi_1 \vee \psi_2$	iff	$\Pi \models \psi_1$ or $\Pi \models \psi_2$
$\Pi \models \neg\psi$	iff	$\Pi \not\models \psi$
$\Pi \models \bigcirc\psi$	iff	$\Pi[1..] \models \psi$
$\Pi \models \psi_1\,\mathcal{U}\,\psi_2$	iff	for some i, $\Pi[i,..] \models \psi_2$, and for all $j < i$ $T, \Pi[j,..] \models \psi_1$

$T, \Pi \models \forall\pi.\varphi$	iff	$T, \Pi[\pi \mapsto t] \models \varphi$ for all $t \in T$
$T, \Pi \models \exists\pi.\varphi$	iff	$T, \Pi[\pi \mapsto t] \models \varphi$ for some $t \in T$
$T, \Pi \models \psi$	iff	$\Pi \models \psi$

Hyperproperties – Relational HyperLTL

The temperature difference between two sensors never exceeds 5 $^\circ$C.

$$\varphi_t = \forall\pi.\,\forall\tau.\,\square\big(|t_\pi - t_\tau| \leq 5\big)$$

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A binary trace predicate that cannot be expressed using atomic propositions.

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Non-interference:
Low-equivalent inputs evaluate to low-equivalent outputs.

$$\varphi_n = \forall\pi_1.\,\forall\pi_2.\,\big(\,\mathrm{in}(\pi_1) =_L \mathrm{in}(\pi_2) \rightarrow \mathrm{out}(\pi_1) =_L \mathrm{out}(\pi_2)\big)$$

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Given a signature $\sigma = (S, \mathrm{ar})$, for $r \in S$,

$$\varphi ::= \forall \pi.\varphi \;\big|\; \exists \pi.\varphi \;\big|\; \psi \qquad \psi ::= r(e, \ldots, e) \;\big|\; \neg\psi \;\big|\; \psi \vee \psi \;\big|\; \bigcirc\psi \;\big|\; \psi\,\mathcal{U}\,\psi \qquad e ::= x_\pi$$

Given a σ-structure $\mathcal{A} = (|\mathcal{A}|, I)$,

$\Pi \models r(e_1, \ldots, e_n)$	iff	$I_{\mathcal{A}}(r)(\llbracket e_1\rrbracket_\Pi, \ldots, \llbracket e_n\rrbracket_\Pi)$
$\Pi \models \psi_1 \vee \psi_2$	iff	$\Pi \models \psi_1$ or $\Pi \models \psi_2$
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$\llbracket x_\pi\rrbracket_\Pi$	$=$	$\Pi(\pi)[0](x)$
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$T, \Pi \models \exists\pi.\varphi$	iff	$T, \Pi[\pi \mapsto t] \models \varphi$ for some $t \in T$
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Monitoring HyperLTL

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- Observation: the world today at 10am

$U_{10} = \{\text{☕☕☕☕}\}$

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- Update: the world at 11am

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φ_u Is there always coffee everywhere at the same time?

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φ_a Is there always coffee somewhere? $U_{10} \rightarrow$ **?**, $U_{11} \rightarrow$ **?**

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Definition

A finite observation $U \in \mathcal{P}_{\mathit{fin}}(\Sigma^*)$ permanently satisfies (resp. violates) φ, if every infinite extension of U satisfies (resp. violates) φ:

$$
\begin{aligned}
U \text{ perm. satisfies } \varphi \quad & \text{iff} \quad \text{all } T \in \mathcal{P}(\Sigma^\omega) \text{ such that } U \preceq T \text{ satisfy } \varphi \\
U \text{ perm. violates } \varphi \quad & \text{iff} \quad \text{all } T \in \mathcal{P}(\Sigma^\omega) \text{ such that } U \preceq T \text{ violate } \varphi
\end{aligned}
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$$\begin{array}{rll} U \text{ perm. satisfies } \varphi & \text{iff} & \text{all } T \in \mathcal{P}(\Sigma^\omega) \text{ such that } U \preceq T \text{ satisfy } \varphi \\ U \text{ perm. violates } \varphi & \text{iff} & \text{all } T \in \mathcal{P}(\Sigma^\omega) \text{ such that } U \preceq T \text{ violate } \varphi \end{array}$$

$$U_{11} = \{☕☕☕☕☕🍵🍵🍵,\ 🍵🍵🍵🍵☕☕☕☕,\ ☕☕🍵\}$$

U_{11} doesn't perm. satisfy $\forall\pi.\forall\tau.\Box(☕_\pi \rightarrow ☕_\tau)$

U_{11} perm. violates $\forall\pi.\forall\tau.\Box(☕_\pi \rightarrow ☕_\tau)$

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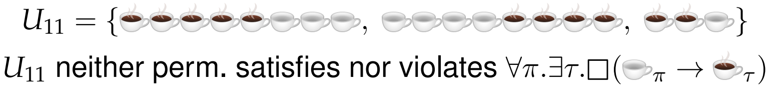
Monitors for HyperLTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation U.



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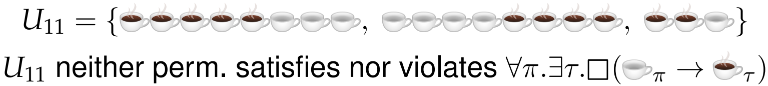

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Monitorability of HyperLTL formulas

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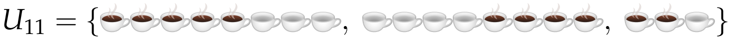

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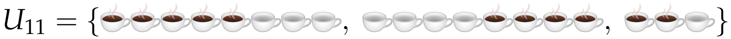

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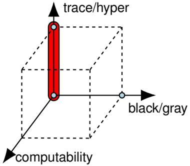
Definition (Agrawal & Bonakdarpour 2016)

A formula φ is *(semantically) monitorable* if every observation U has an extension $V \succeq U$, such that V perm. satisfies φ or V perm. violates φ.

HyperLTL – Summary



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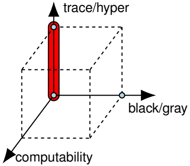

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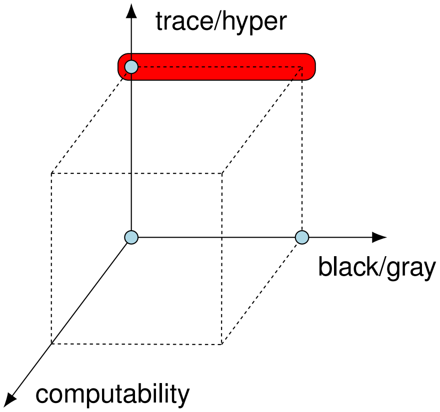
[1] S. Agrawal and B. Bonakdarpour. *Runtime Verification of k-Safety Hyperproperties in HyperLTL.* CSF'16, IEEE CS Press, 2016.

[9] K. Havelund and D. Peled. *Runtime Verification: From Propositional to First-Order Temporal Logic.* RV'18, Springer, 2018.

[8] C. Hahn. *Algorithms for Monitoring Hyperproperties.* RV'19, Springer, 2019.

Gray-box monitoring (of hyperproperties)



Why is φ_a not monitorable?

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No finite U permanently satisfies or violates $\varphi_a = \forall\pi.\exists\tau.\square($$_\pi \rightarrow$ $_\tau)$.

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This theorem can be generalized to all formulas $\varphi = \forall\pi.\exists\tau.\Box P(\pi,\tau)$ where P is

- a binary (non-temporal) predicate,
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OK, but let's have a closer look at this proof…

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When monitoring hyperproperties, we'd like to take into account
some information about the system
(gray-box monitoring).

Gray-box monitoring of HyperLTL properties

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation U.

Definition

A finite observation $U \in \mathcal{P}_{fin}(\Sigma^*)$ permanently satisfies (resp. violates) φ, if every infinite extension of U satisfies (resp. violates) φ:

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$\mathcal{S} = \{T \in \mathcal{P}(\Sigma^\omega) \mid |T| = 3\}$ $U = \{$  ,  ,  $\}$

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Gray-box monitoring in general

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation O of a system $\mathcal{S}$.

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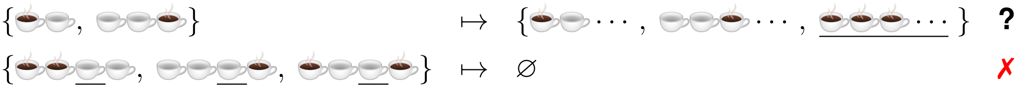

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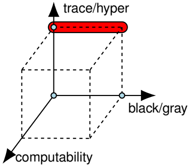
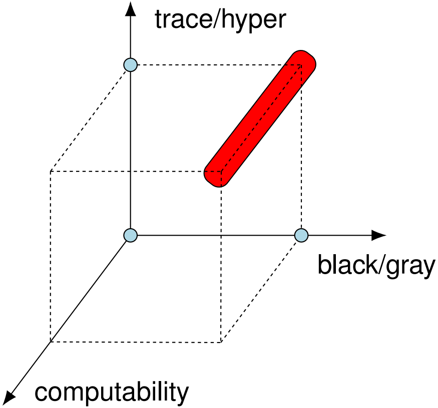
- Perfect monitors can be constructed for some formulas.
  - For example, for formulas without quantifier alternations (as for black-box).
  - But also for $\forall^+\exists^+$-formulas when $\mathcal{S}$ imposes enough constraints.
- Monitorability of formulas depends on set of valid system behaviors $\mathcal{S}$.
  - For example, $\forall^+\exists^+$-properties are monitorable for some choices of $\mathcal{S}$.
  - We will see a more interesting example later…



Undecidable hyperproperties



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A monitor for a property φ and a system $\mathcal{S}$ is a computable function $M_{\varphi,\mathcal{S}} : \mathcal{O}\{\checkmark, \text{✗}, \textbf{?}\}$ that decides a verdict for φ given a finite O.

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$$\mathcal{S} = \{t \in \Sigma^\omega \mid t_i = \text{ the state of } T \text{ after } i \text{ steps}\}, \qquad \varphi = \Diamond\text{halt}.$$

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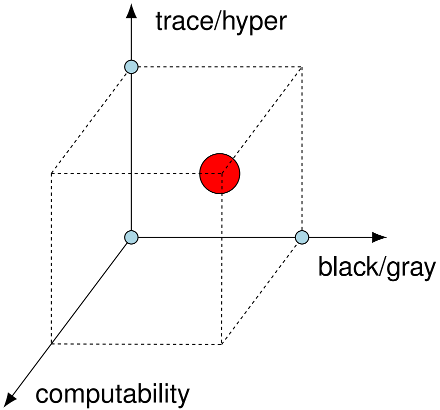
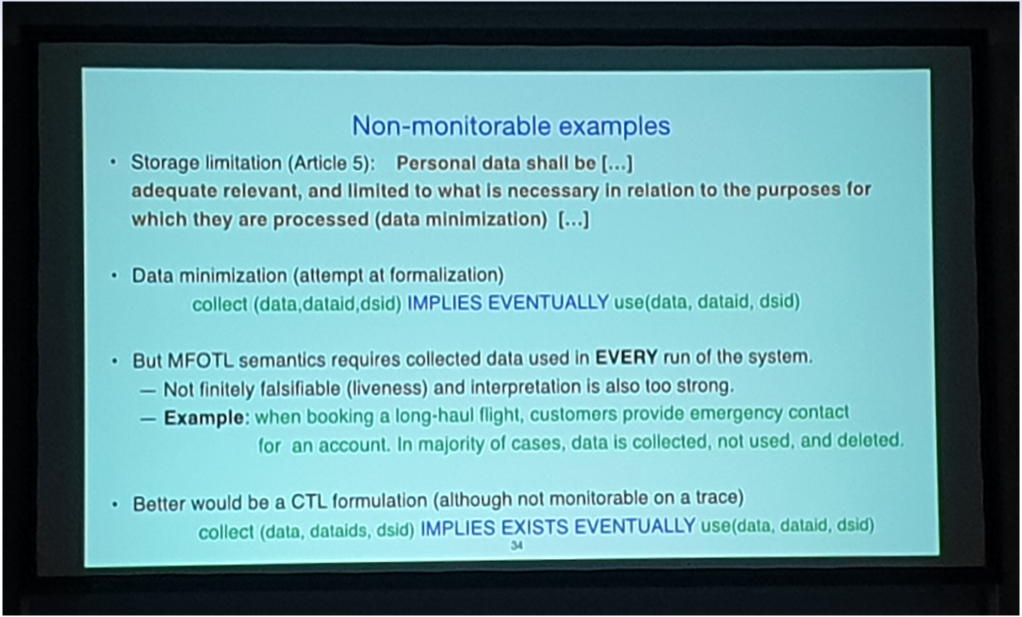
- $\Rightarrow$ φ is monitorable in $\mathcal{S}$;
- $\Rightarrow$ *there is a sound monitor $M_{\varphi,\mathcal{S}}$ that only answers ✓ or **?**!*

Case study: distributed data minimality





Slide by David Basin, *Can we Verify GDPR Compliance?*, RV’19 keynote.

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- privacy property (GDPR)

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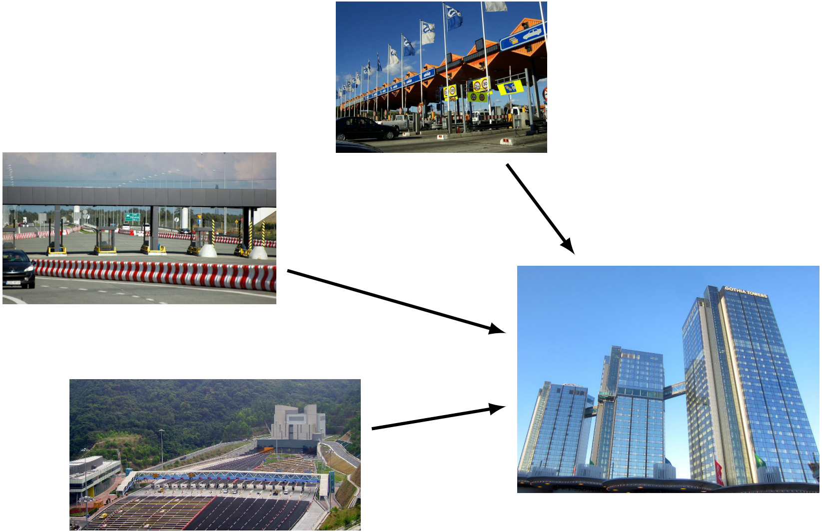

Photos by Rauenstein, Radosław Drożdżewski, Chong Fat, Hesekiel (Wikipedia).

DDM example: toll road

```
class Toll {
  int rate(int hour, int passengers) {
    int r;                                            // standard rates:
    if (hour >= 9 && hour <= 17) { r = 90; }         //   - daytime
    else                          { r = 70; }         //   - nighttime
    if (passengers > 2) { r = r - (r / 5); }         // carpool: 20% off
    return r;
  }

  int fee(int t1, int t2, int t3, int p) {
    int r1 = rate(t1, p);               // rates at each toll station
    int r2 = rate(t2, p);
    int r3 = rate(t3, p);
    int f1 = max(r1, r2) * 4;           // fees per road section
    int f2 = max(r2, r3) * 7;
    return f1 + f2;                     // total fee
  }
}
```

Case study: distributed data minimality

- Distributed data minimality (DDM)
  - privacy property (GDPR)
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  - $\forall\forall\exists\exists$-hyperproperty

$$\varphi_i \;=\; \forall\pi.\forall\pi'.\exists\tau.\exists\tau'.\ \neg\,\mathrm{same}_i(\pi,\pi') \rightarrow \begin{pmatrix} \mathrm{same}_i(\pi,\tau) \wedge \mathrm{same}_i(\pi',\tau') \,\wedge \\ \mathrm{almost}_i(\tau,\tau') \wedge \neg\,\mathrm{output}(\tau,\tau') \end{pmatrix}$$

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Yet, we have a monitor [11]. . .

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Yet, we have a monitor [11]. . .

here's how. . .

Distributed data minimality

Definition (Antignac, Sands & Schneider, 2017)

A function f is distributed data-minimal (DDM) if, for all input positions k and all $x, y \in I_k$ such that $x \neq y$, there is some $z \in I$, such that $f(z[k \mapsto x]) \neq f(z[k \mapsto y])$.

Distributed data minimality

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$$\varphi_{\textsf{dm}} \;=\; \bigwedge_{i=1}^{n} \varphi_i, \qquad \Sigma_f^{\#} = \{(x,y) \mid f(x) = y\}, \qquad \mathcal{S}_f = \mathcal{P}(\Sigma_f^{\#})$$

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- Set of observable behaviors $\mathcal{O} = \Sigma_f^{\#}$ are valid function applications.

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- Set of observable behaviors $\mathcal{O} = \Sigma_f^{\#}$ are valid function applications.
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Using the generalized framework

- Set of observable behaviors $\mathcal{O} = \Sigma_f^{\#}$ are valid function applications.
- Not black-box monitorable, but gray-box monitorable (thanks to $\mathcal{S}$).

A sound monitor for distributed data minimality

$$\varphi_i \;=\; \forall\pi.\forall\pi'.\exists\tau.\exists\tau'.\,\neg\,\mathrm{same}_i(\pi,\pi') \rightarrow \begin{pmatrix} \mathrm{same}_i(\pi,\tau) \wedge \mathrm{same}_i(\pi',\tau') \,\wedge \\ \mathrm{almost}_i(\tau,\tau') \wedge \neg\,\mathrm{output}(\tau,\tau') \end{pmatrix}$$

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We build a monitor

$$M_{\mathsf{dm}}(U) = \begin{cases} \textbf{?} & \text{if } f(u_{\mathit{in}}) \neq u_{\mathit{out}} \text{ for some } u \in U, \\ \textbf{?} & \text{if } \bigwedge_{i=1}^{n} \bigwedge_{u,u' \in U} N_{f,i}(\mathrm{proj}_i(u_{\mathit{in}}), \mathrm{proj}_i(u'_{\mathit{in}})) \neq \text{✗}, \\ \text{✗} & \text{otherwise.} \end{cases}$$

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using an oracle $N_{f,i}(x,y)$ (implemented as symbolic execution + SMT solver):

$$N_{f,i}(x,y) = \begin{cases} ✓ \text{ or } \textbf{?} & \text{if } \exists z \in I.\, f(z[i \mapsto x]) \neq f(z[i \mapsto y]), \\ ✗ \text{ or } \textbf{?} & \text{otherwise.} \end{cases}$$

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The monitor is sound but not perfect.

Please do try this at home!

`https://github.com/sstucki/minion/`

Thank you!

Coauthors

- César Sánchez, IMDEA SW
- Borzoo Bonakdarpour, ISU
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institute imdea software

IOWA STATE UNIVERSITY

Checkout the `minion` monitor for data minimality

`https://github.com/sstucki/minion/`

Backup slides

Trace properties – LTL

$\varphi_s = \Box ☕$ $\qquad$ $\varphi_l = \Diamond ☕$ $\qquad$ $\varphi_r = \Box\Diamond ☕$

Trace properties – LTL

$\varphi_s = \square$☕ $\qquad \varphi_l = \diamond$☕ $\qquad \varphi_r = \square\diamond$☕

$t_1 =$  $\cdots$ $\qquad t_1 \models \varphi_s \qquad t_1 \models \varphi_l \qquad t_1 \models \varphi_r$

$t_2 =$  $\cdots$ $\qquad t_2 \not\models \varphi_s \qquad t_2 \models \varphi_l \qquad t_2 \not\models \varphi_r$

$t_3 =$  $\cdots$ $\qquad t_3 \not\models \varphi_s \qquad t_3 \models \varphi_l \qquad t_3 \models \varphi_r$

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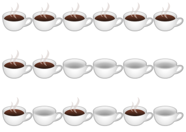

$t_1 = \cdots \qquad t_1 \models \varphi_s \qquad t_1 \models \varphi_l \qquad t_1 \models \varphi_r$

$t_2 = \cdots \qquad t_2 \not\models \varphi_s \qquad t_2 \models \varphi_l \qquad t_2 \not\models \varphi_r$

$t_3 = \cdots \qquad t_3 \not\models \varphi_s \qquad t_3 \models \varphi_l \qquad t_3 \models \varphi_r$

$\varphi ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \bigcirc\varphi \mid \varphi\,\mathcal{U}\,\varphi \qquad \Diamond\varphi \equiv \text{true}\ \mathcal{U}\ \varphi \qquad \Box\varphi \equiv \neg\Diamond\neg\varphi$

$t \models p$	iff	$p \in t[0]$
$t \models \neg\varphi$	iff	$t \not\models \varphi$
$t \models \varphi_1 \vee \varphi_2$	iff	$t \models \varphi_1$ or $t \models \varphi_2$
$t \models \bigcirc\varphi$	iff	$t[1,..] \models \varphi$
$t \models \varphi_1\,\mathcal{U}\,\varphi_2$	iff	for some i, $t[i,..] \models \varphi_2$ and for all $j < i$, $t[j,..] \models \varphi_1$

Monitoring LTL

$\varphi_s = \Box$☕ $\quad \varphi_l = \Diamond$☕ $\quad \varphi_r = \Box\Diamond$☕

Monitoring LTL

$\varphi_s = \square$☕ $\varphi_l = \lozenge$☕ $\varphi_r = \square\lozenge$☕

- Observation: the world today at 10am

  $u_{10} =$ ☕☕☕☕

Monitoring LTL

$$\varphi_s = \Box ☕ \qquad \varphi_l = \Diamond ☕ \qquad \varphi_r = \Box\Diamond ☕$$

- Observation: the world today at 10am

  $u_{10} =$ ☕☕☕☕

- Update: the world at 11am

  $u_{11} =$ ☕☕☕☕☕☕☕☕

Monitoring LTL

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- Observation: the world today at 10am

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- Update: the world at 11am

$u_{11} =$ ☕☕☕☕☕

φ_s Is there always coffee?

Monitoring LTL

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- Update: the world at 11am

  $u_{11} =$ ☕☕☕☕☕

φ_s Is there always coffee? $\qquad u_{10} \;\rightarrow\;$ **?**

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  $u_{10} =$ ☕☕☕☕

- Update: the world at 11am

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φ_s Is there always coffee? $\qquad u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ ✗

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- Update: the world at 11am

  $u_{11} =$ ☕☕☕☕☕☕☕☕

φ_s	Is there always coffee?	$u_{10} \rightarrow$ **?** , $u_{11} \rightarrow$ ✗
φ_l	Is there eventually coffee?	$u_{10} \rightarrow$ ✓, $u_{11} \rightarrow$ ✓

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$u_{11} =$ ☕☕☕☕☕☕☕☕

φ_s	Is there always coffee?	$u_{10} \to$ **?** , $u_{11} \to$ ✗
φ_l	Is there eventually coffee?	$u_{10} \to$ ✓, $u_{11} \to$ ✓
φ_r	Is there always eventually coffee?	$u_{10} \to$ **?** , $u_{11} \to$ **?**

Monitoring LTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), at runtime.

Monitoring LTL

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

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Definition

A finite observation u permanently satisfies (resp. violates) φ, if every infinite extension of u satisfies (resp. violates) φ:

$$\begin{array}{rll} u \text{ perm. satisfies } \varphi & \text{iff} & \text{all } t \in \Sigma^\omega \text{ such that } u \preceq t \text{ satisfy } \varphi \\ u \text{ perm. violates } \varphi & \text{iff} & \text{all } t \in \Sigma^\omega \text{ such that } u \preceq t \text{ violate } \varphi \end{array}$$

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u perm. violates φ iff all $t \in \Sigma^\omega$ such that $u \preceq t$ violate φ

$u_{11} =$ 

u_{11} doesn't perm. satisfy $\Box$☕ | u_{11} perm. violates $\Box$☕

u_{11} perm. satisfies $\Diamond$☕ | u_{11} doesn't perm. violate $\Diamond$☕

u_{11} neither perm. satisfies nor violates $\Box\Diamond$☕

Monitors for LTL

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The monitor M_φ is sound if

u perm. satisfies φ if $M_\varphi(u) = ✓$, u perm. violates φ if $M_\varphi(u) = ✗$

Monitors for LTL

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The monitor M_φ is sound if

$$u \text{ perm. satisfies } \varphi \quad \text{if} \quad M_\varphi(u) = ✓, \qquad u \text{ perm. violates } \varphi \quad \text{if} \quad M_\varphi(u) = ✗$$

The monitor M_φ is perfect if, additionally,

$$M_\varphi(u) = ✓ \text{ if } u \text{ perm. satisfies } \varphi, \quad M_\varphi(u) = ✗ \text{ if } u \text{ perm. violates } \varphi,$$
$$M_\varphi(u) = \textbf{?} \text{ o/w.}$$

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A monitor for a property φ is a computable function $M_\varphi\colon \Sigma^* \to \{✓, ✗, \textbf{?}\}$ that decides a verdict for φ given a finite u.

The monitor M_φ is sound if

u perm. satisfies φ if $M_\varphi(u) = ✓$, u perm. violates φ if $M_\varphi(u) = ✗$

The monitor M_φ is perfect if, additionally,

$M_\varphi(u) = ✓$ if u perm. satisfies φ, $M_\varphi(u) = ✗$ if u perm. violates φ,
$M_\varphi(u) = \textbf{?}$ o/w.

Fact: every LTL formula has a perfect monitor.

Monitorability of LTL formulas

Monitoring: decide whether a given property φ is permanently satisfied (✓), violated (✗), or neither (**?**), given a finite observation u.

$\varphi_r = \Box\Diamond$☕ $u_{11} =$ 

u_{11} doesn't perm. satisfy φ_r u_{11} doesn't perm. violate φ_r

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Observation: There is no u that permanently satisfies or violates φ_r.

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There's no point in monitoring φ_r!

Definition (Pnueli & Zaks 2006)

A formula φ is *(semantically) monitorable* if every observation u has an extension $v \succeq u$, such that either v perm. satisfies φ or v perm. violates φ.

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